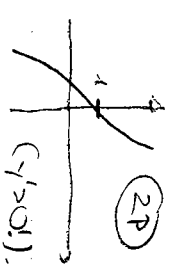
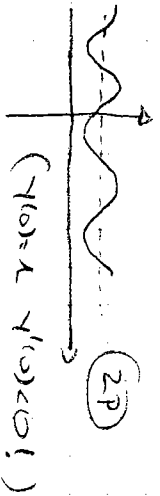


4a) $y = x^3 + x^2 + 3x + C$ (4P)  (2P)

$y(0) = C \stackrel{!}{=} 1 \Rightarrow C = 1$ (2P) $(y' > 0)$

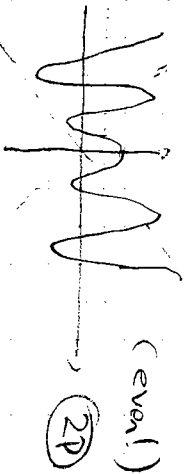
b) $y = -\frac{2}{3} \cos(3x) - \frac{1}{3} \sin(5x) + C_1$ (4P)

$y(0) = -\frac{2}{3} + C \stackrel{!}{=} 1 \Rightarrow C_1 = \frac{5}{3}$ (2P)



c) $y = \int x \cos(3x) dx = \frac{x}{3} \sin(3x) - \frac{1}{9} \int \sin(3x) dx$
 $= \frac{x}{3} \sin(3x) + \frac{1}{9} \cos(3x) + C$ (4P)

$y(0) = \frac{1}{9} + C \stackrel{!}{=} 1 \Rightarrow C = \frac{8}{9}$ (2P)



d) $\int e^{-x} \cos x dx = e^{-x} \sin x - \int (-e^{-x}) \sin x dx$

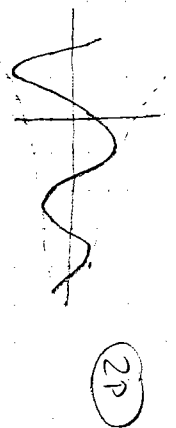
$= e^{-x} \sin x + (e^{-x} (-\cos x) - \int (-e^{-x}) (-\cos x) dx)$

$= e^{-x} (\sin x - \cos x) - \int e^{-x} \cos x dx$

$\Rightarrow 2 \int e^{-x} \cos x dx = e^{-x} (\sin x - \cos x) + C_1$ (2P)

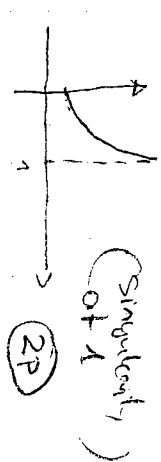
$\Rightarrow y = e^{-x} (\sin x - \cos x) + C$ (2P)

$y(0) = -1 + C \stackrel{!}{=} 1 \Rightarrow C = 2$ (2P)



e) $y = 3 \int \frac{dx}{(1-x)(1+x)} = 3 \int \frac{\frac{1}{2}}{(1-x)(1+x)} dx$

$= -\ln|1-x| + \ln|1+x| + C_1 = \ln \left| \frac{1+x}{1-x} \right| + C_1$ (4P)

$y(0) = \ln 1 + C_1 = 1$
 $\Rightarrow C = 1 - \ln 1$ (2P)  (4P)

5.)

I) $y' = 2x = 2 \sqrt{(1+x^2)-1} = 2 \sqrt{y-1} \Rightarrow D$ (4P)

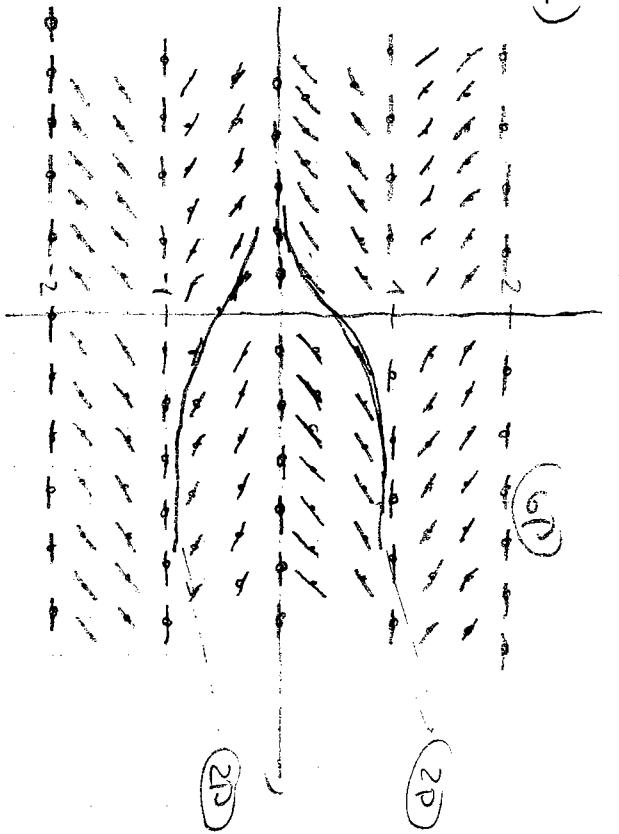
II) $y' = \frac{1}{(1-x)} + \frac{x}{(1-x)^2} = \frac{y}{x} + \frac{y^2}{x} = \frac{y(1+y)}{x} \Rightarrow C$ (4P)

III) $y' = \frac{1}{2} \frac{2x}{1+x^2} = \frac{x}{y-1} \Rightarrow A$ (4P)

IV) $y' = e^x + x e^x = (1+x) e^x = (1+x) \frac{y}{x} \Rightarrow B$ (4P)

V) $y' = -\sin(x-1) = -\sqrt{1-y^2}$
 (in some domain) $\Rightarrow E$ (4P)

6a)



B) $(1 + \frac{1}{n})^n \rightarrow e = 0 \quad (1 + \frac{1}{n^2})^{n^2} \rightarrow e$

$\lim_{n \rightarrow \infty} n^2 \ln(1 + \frac{1}{n^2}) = 1$ (4P)

$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

$\Rightarrow 0 = 0 \cdot 1 = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \lim_{n \rightarrow \infty} n^2 \ln(1 + \frac{1}{n^2})$

$= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot n^2 \ln(1 + \frac{1}{n^2})$

$= \lim_{n \rightarrow \infty} \ln(1 + \frac{1}{n^2})^n = \ln \lim_{n \rightarrow \infty} (1 + \frac{1}{n^2})^n$

$\lim_{n \rightarrow \infty} (1 + \frac{1}{n^2})^n = 1$ (6P)

$\ln(1) = 0$

6b)

